Nuclear Structure from Gamma-Ray Spectroscopy

2019 Postgraduate Lectures

Lecture 7: Broken Symmetries

Broken Symmetries

- Reflection Asymmetry: Octupole Bands
- Handedness: Chiral Bands
- Magnetic Rotation: Shears Bands
- Transitional Nuclei: Critical Points

Reflection Asymmetry

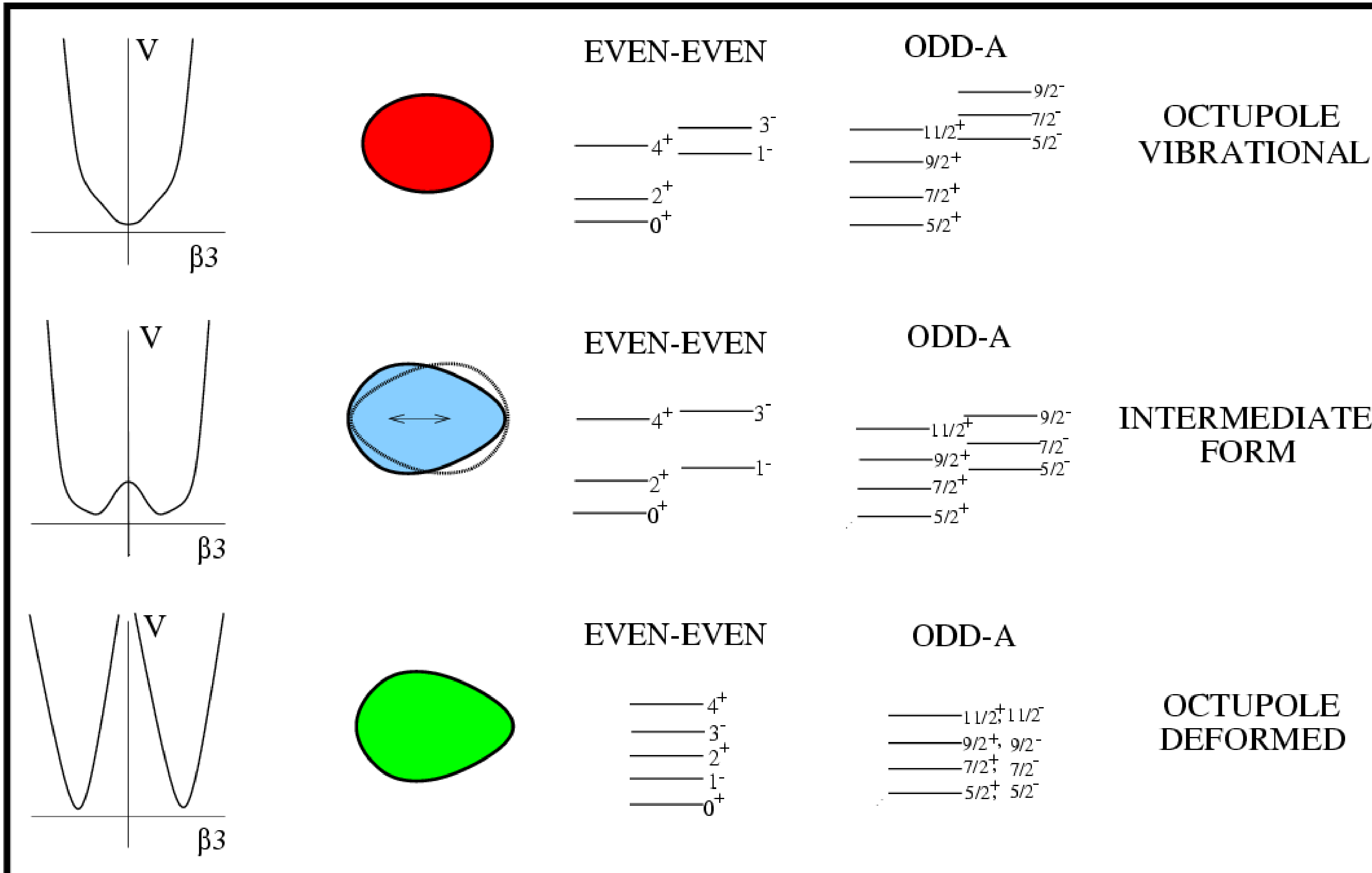
- If a nucleus is 'reflection asymmetric' (i.e. the odd multipole deformation parameters are non-zero, e.g. $\beta_3 \neq 0$ is the most important) then the nuclear wavefunction in its intrinsic frame is not an eigenvalue of the parity operator:

$$\Psi^2(x, y, z) \neq \Psi^2(-x, -y, -z)$$

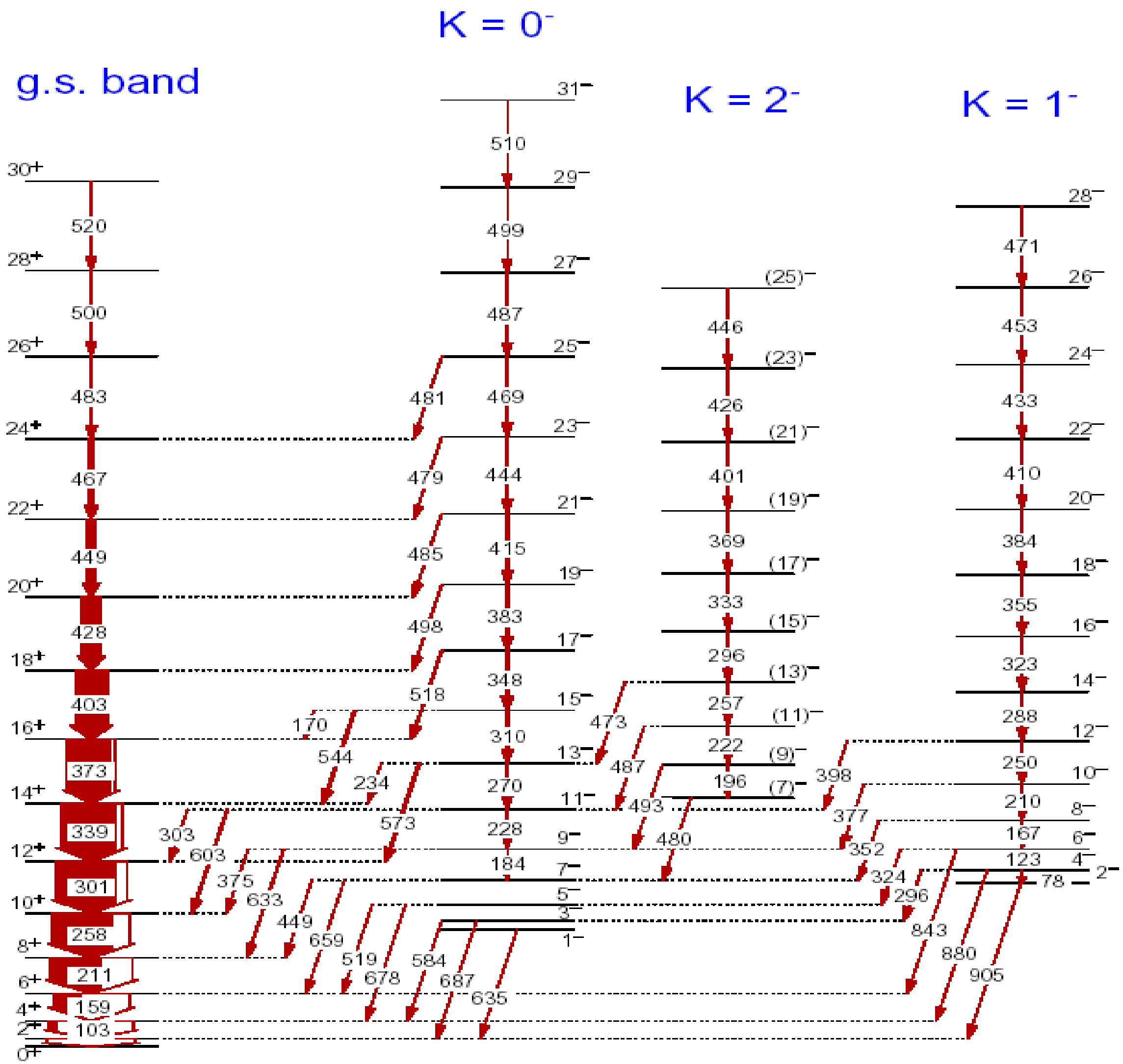
- If $\beta_3 \neq 0$ for a nucleus it is said to possess octupole deformation

- The deformation can however be static, $\langle \beta_3 \rangle \neq 0$, or dynamic, $\langle \beta_3 \rangle = 0$ (oscillating octopule shape)

Octupole Band Structures



Octupole Vibrations in ^{238}U



- This nucleus shows three octupole vibrational bands with different K values

Parity Splitting

- For a static octupole shape, the negative parity states are interleaved (midway between) with the positive parity states

- A measure of such a feature is the 'parity splitting', defined as:

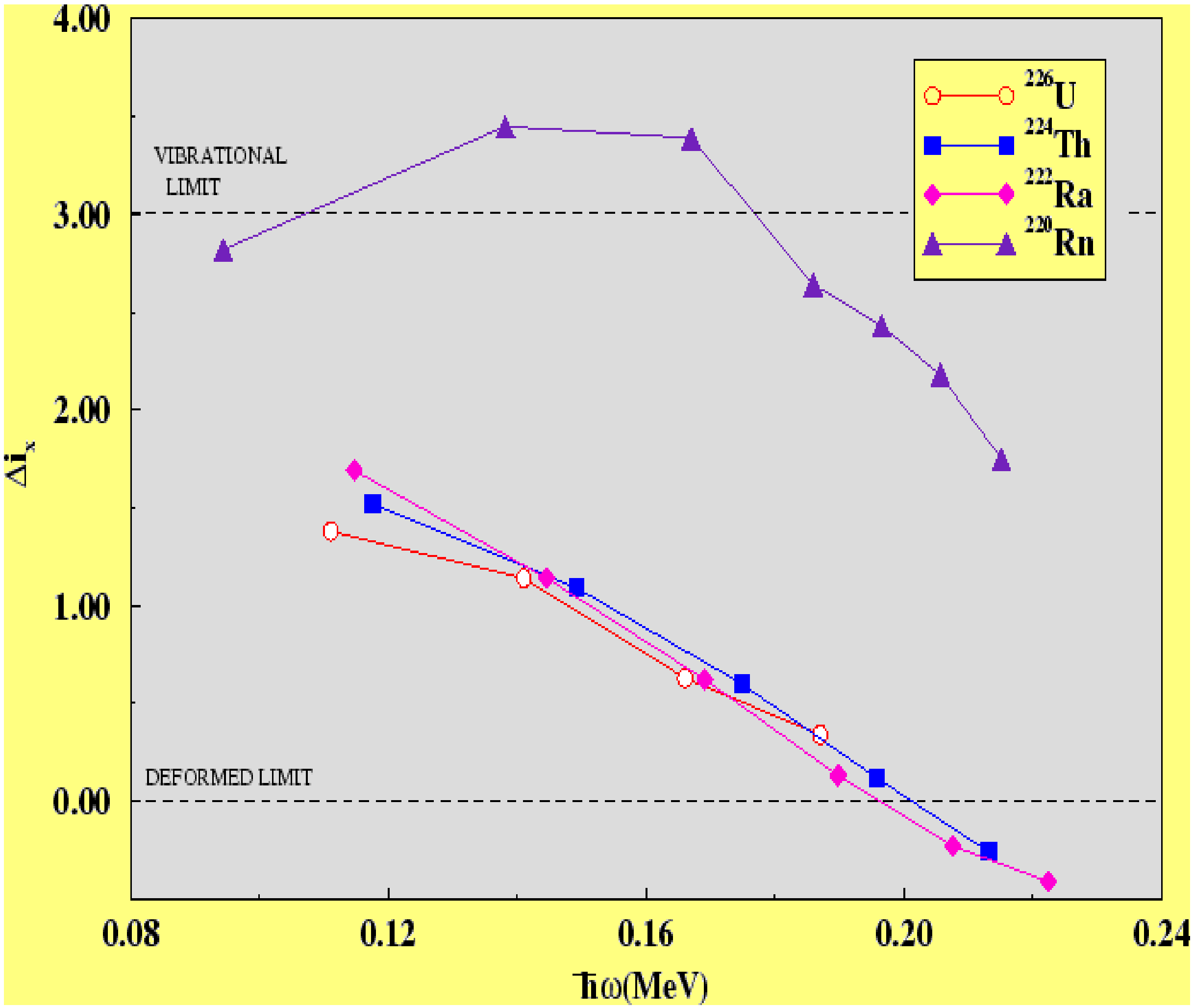
$$\delta E = E(I)^- - \tfrac{1}{2}[\, E(I+1)^+ + E(I-1)^+ \,]$$

- This quantity generally decreases towards zero with increasing spin and suggests that rotation may stabilise the octupole shape

- A similar quantity is the difference in alignment:

$$\Delta i_x = i_x^- - i_x^+$$

Octupole Vibration or Deformed?



- For an octupole vibrational phonon coupled to the positive-parity states:

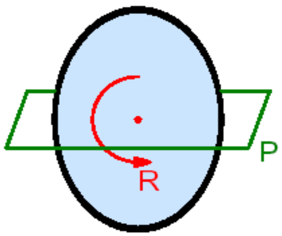
$$\Delta i_x = 3\,\hbar$$

- For a static octupole deformation:

$$\Delta i_x = 0$$

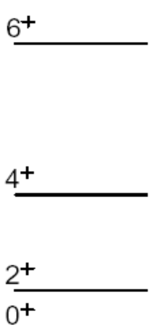
Reflection (A)symmetry

K = angular momentum projection on symmetry axis



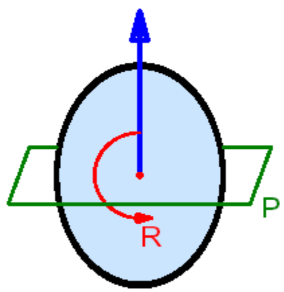
K = 0

P: parity (reflection)
R: rotation by 180°
T: time reversal



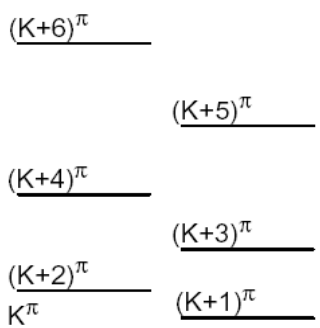
K = 0

1 band



K > 0

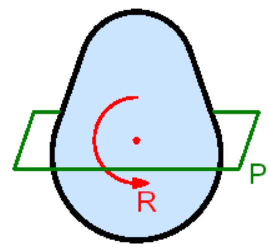
P: parity (reflection)
RT: rotation by 180° AND time reversal (which reverses K)



K > 0

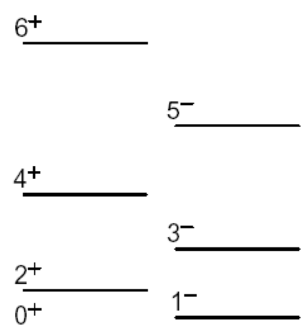
2 bands

K = angular momentum projection on symmetry axis



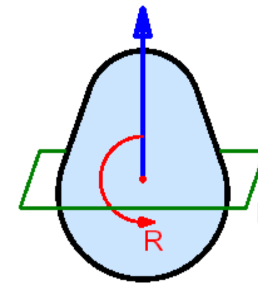
K = 0

RP: rotation & reflection
T: time reversal



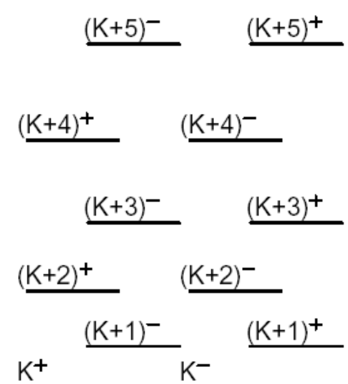
K = 0

2 bands



K > 0

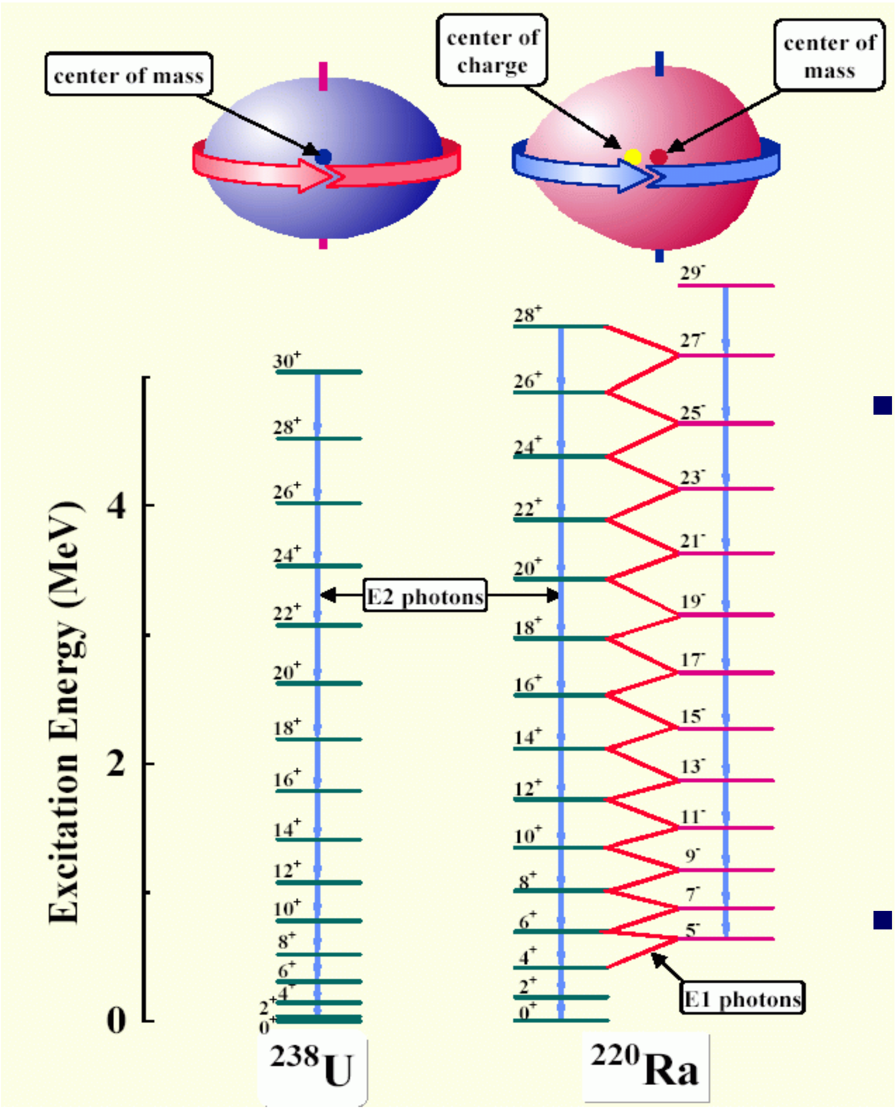
RPT: need all three operations

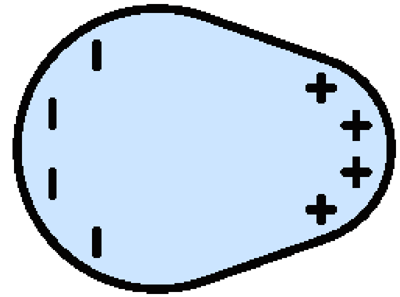


K > 0

4 bands

Electric Dipole Moment





- In a nucleus with octupole deformation, the centre of mass and centre of charge tend to separate, creating a non-zero electric dipole moment
- Bands of opposite parity connected by strong E1 transitions occur

Enhanced E1 Transitions

- In heavy nuclei, E1 strengths typically lie between 10^{-4} and 10^{-7} Wu
- In nuclei with octupole deformation, the E1 strengths can be much higher: 10^{-3} – 10^{-2} Wu
- The intrinsic dipole moment of an octupole deformed nucleus is:

$$D_0 = C_{LD} \, A \, Z \, e \, \beta_2 \, \beta_3$$

with the liquid drop constant $C_{LD} = 0.0007$ fm

- In a Strutinsky type approach, macroscopic and microscopic effects can be considered and:

$$D = D_{macro} + D_{shell}$$

Experimental Dipole Moments

- Experimental values of D_0 can be obtained by measuring $B(E1)/B(E2)$ ratios, related simply to γ-ray energies and intensities

- The $B(E1)$ reduced transition rate is:

$$B(E1;I \to I-1) = (3/4\pi)\, e^2 D_0^2\, |\langle\, I_i\, K_i\, 1\, 0 \mid I_f\, K_f\, \rangle|^2$$

- The $B(E2)$ reduced transition rate is:

$$B(E2;I \to I-2) = (5/16\pi)\, e^2 Q_0^2\, |\langle\, I_i\, K_i\, 2\, 0 \mid I_f\, K_f\, \rangle|^2$$

- Hence if Q_0 is known (e.g. from the quadrupole deformation β_2) then a value for D_0 can be extracted, i.e:

$$D_0 = \sqrt{[5B(E1)/16B(E2)]}\; Q_0$$

Simplex Quantum Number

- The only symmetries for a rotating reflection symmetric nucleus are parity p and signature r

- For a reflection asymmetric shape (e.g. octupole) these are no longer good quantum numbers but the nucleus is invariant with respect to a combination of rotation of 180° about the x axis ($R(\pi)$) and change of parity (P)

- The 'simplex' operator is defined as:

$$S = P\,R(\pi)^{-1}$$

with eigenvalues: $s = -pr = \pm i\ ,\ \pm 1$ $\quad (p = s\exp[i\pi I])$

Parity Doublets

- For $K \neq 0$, four $\Delta I = 2$ (E2) bands are formed based on states with $K^{\pm}$ and $(K+1)^{\pm}$

- The simplex quantum number can be used to classify these structures

- For an even-even nucleus:

  $s = +1$ describes states (0^+), 1^-, 2^+, 3^-, 4^+...

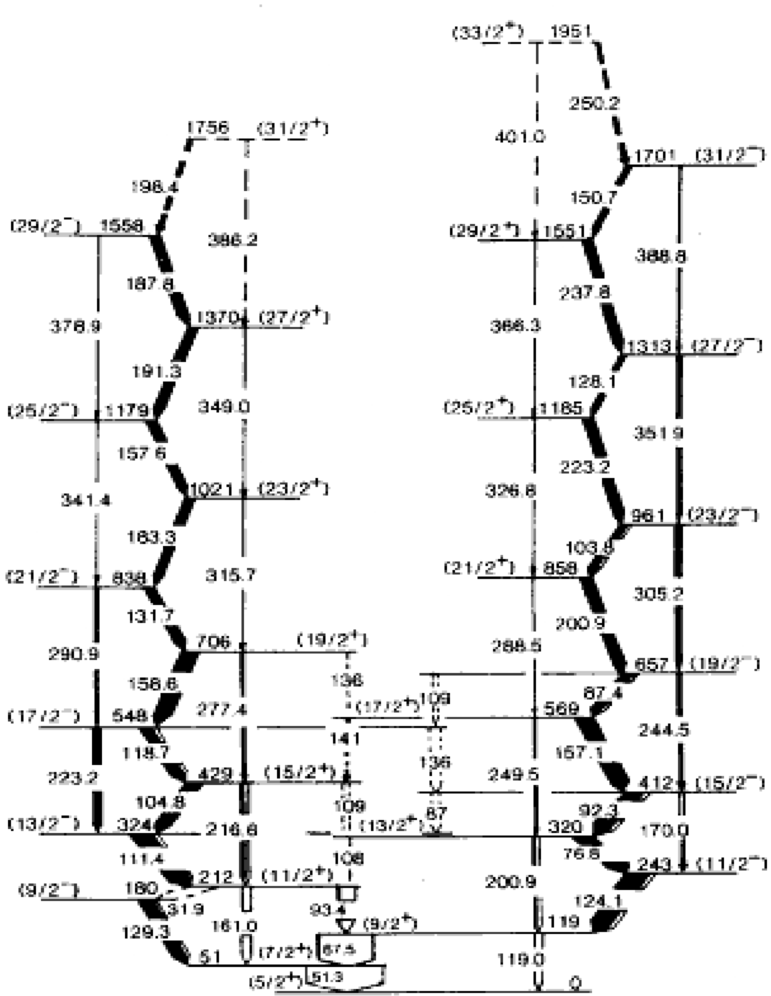
  $s = -1$ describes states (0^-), 1^+, 2^-, 3^+, 4^-...

- For an odd-A nucleus:

  $s = +i$ describes states $1/2^+$, $3/2^-$, $5/2^+$, $7/2^-$,...

  $s = -i$ describes states $1/2^-$, $3/2^+$, $5/2^-$, $7/2^+$,...

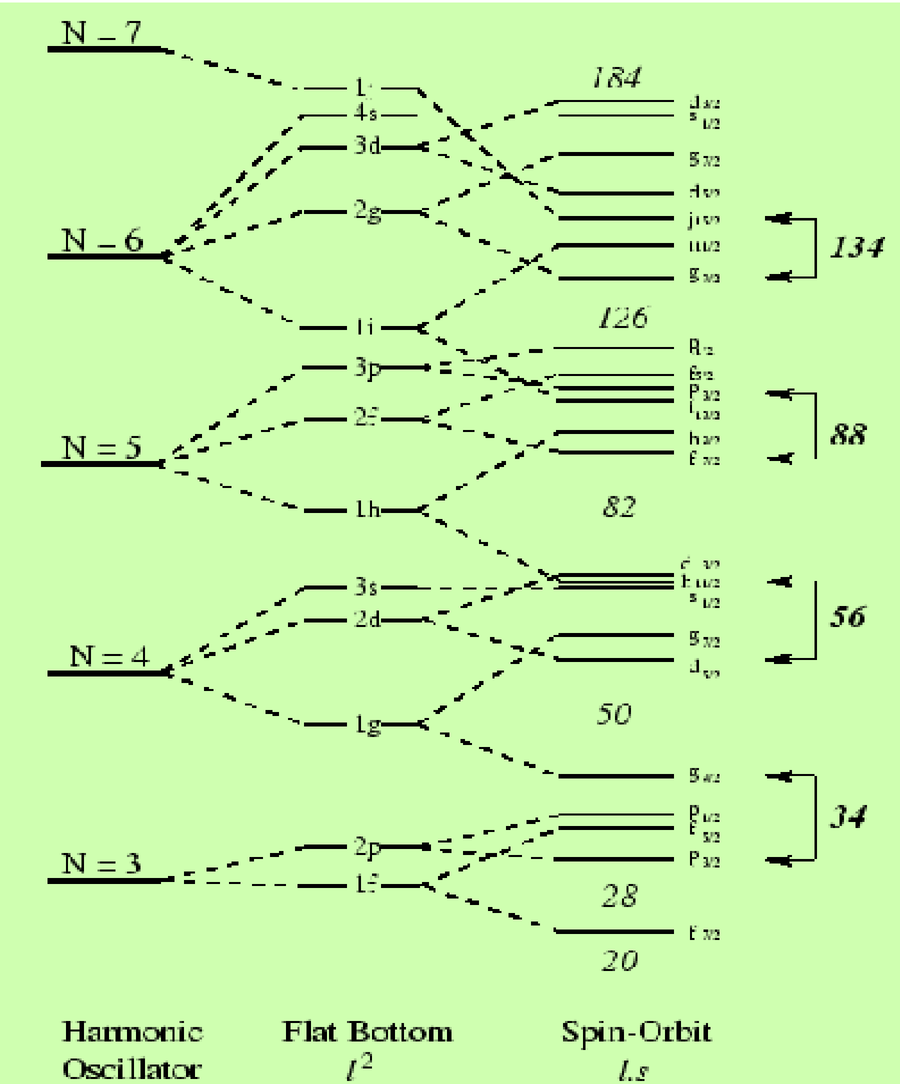
Parity Doublets in ^{223}Th



s = -i   s = +i

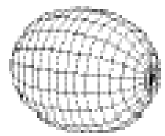
- The nucleus ^{223}Th shows parity doublets
- The two $\Delta I = 2$ bands, shown to the left, are connected by strong E1 transitions and have simplex $s = -i$
- The two $\Delta I = 2$ bands, to the right, have simplex $s = +i$
- M1 transitions also connect some of the bands

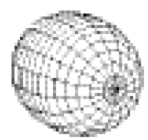
Octupole Magic Numbers



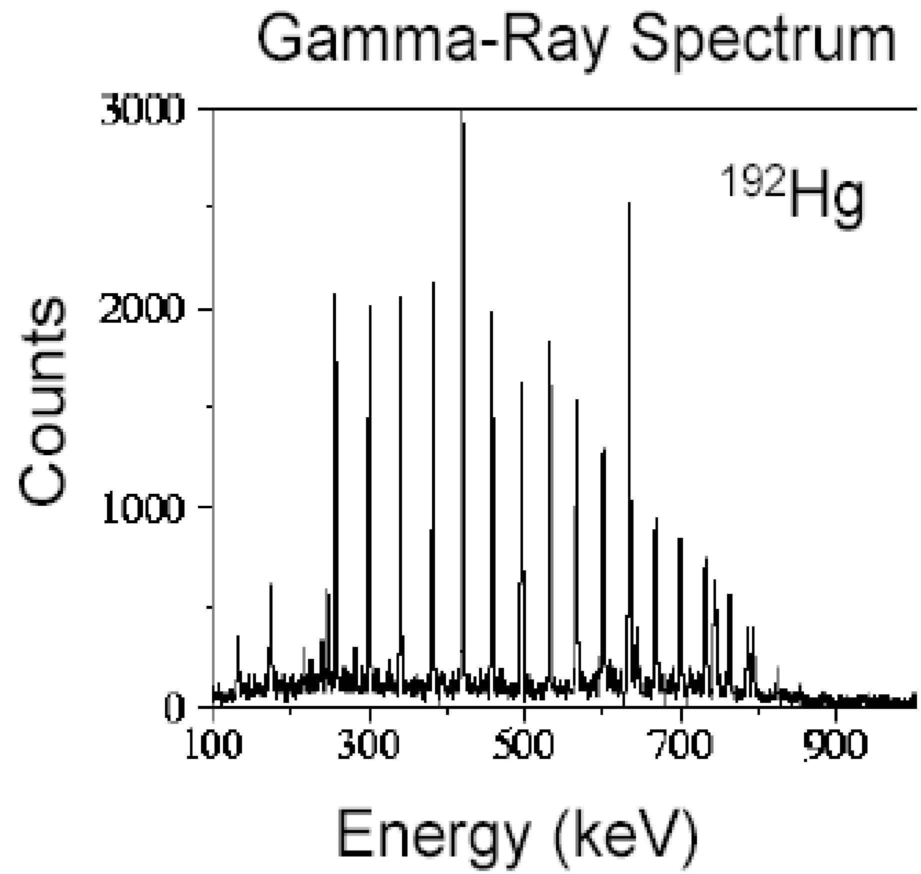
- Octupole correlations occur between orbitals which differ in both orbital (ℓ) and total (j) angular momenta by 3
- Magic numbers occur at 34, 56, 88 and 134
- Nuclei with both proton and neutron numbers close to these are the best candidates to show octupole effects

Rotational Invariance

$|\Psi\rangle =$ |  $\rangle$

$R(\omega)|\Psi\rangle =$ |  $\rangle$

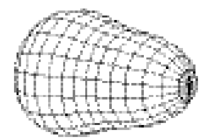
$$R(\omega)|\Psi\rangle \neq |\Psi\rangle$$

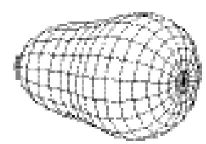


- From Kris Starosta (Michigan State University)

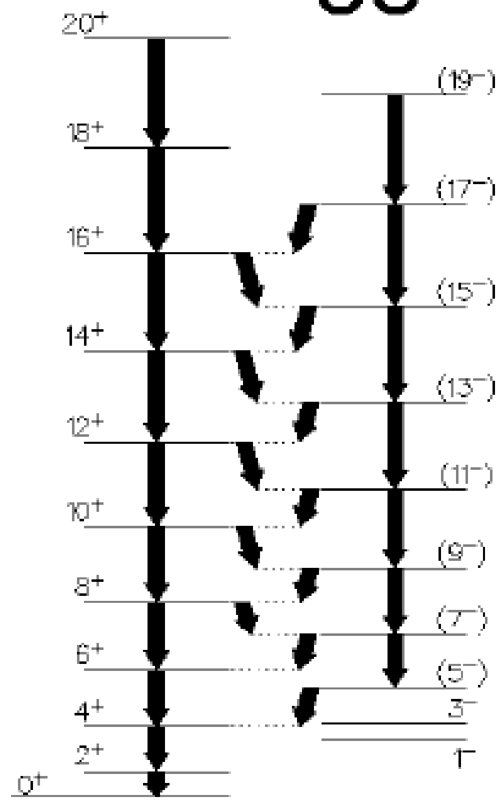
Space Inversion Invariance

$^{222}_{88}Ra$

$|\Psi\rangle = |$  $\rangle$

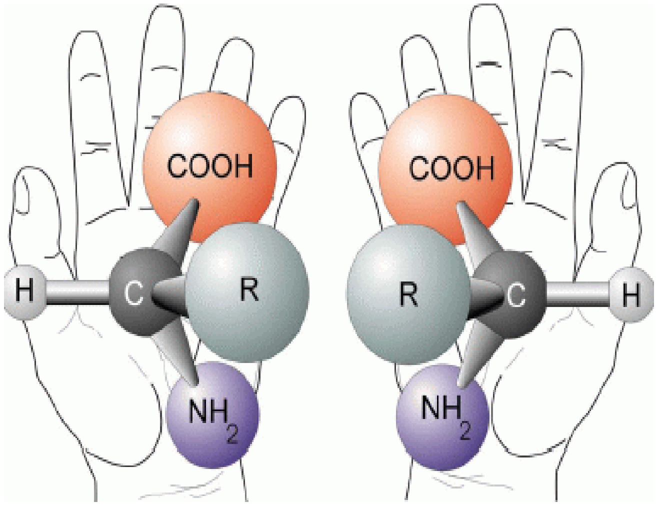
$P|\Psi\rangle = |$  $\rangle$

$$P|\Psi\rangle \neq |\Psi\rangle$$

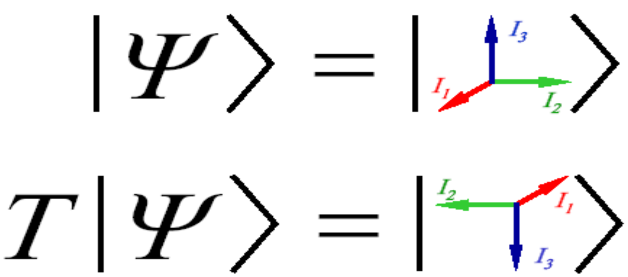


- From Kris Starosta (Michigan State University)

Chirality (Handedness)



- 'I call any geometric figure, or group of points, chiral, and say it has chirality, if its image in a plane mirror, ideally realised, cannot be brought to coincide with itself' Lord Kelvin 1904

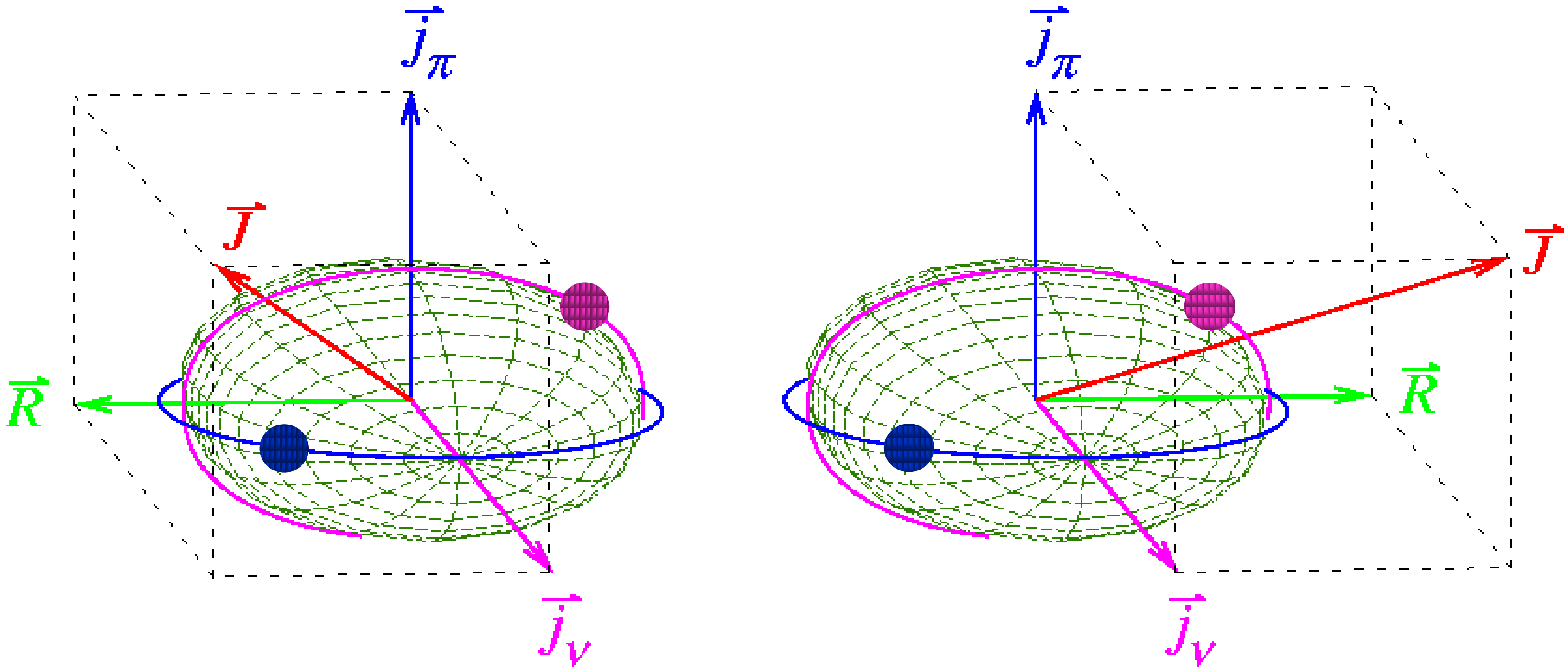


$$T|\Psi\rangle \neq |\Psi\rangle$$

- Examples of chiral systems are found throughout nature and in several disciplines of science

- Axial vectors of angular momenta systems of opposite chirality are related by time reversal

Chiral Geometry



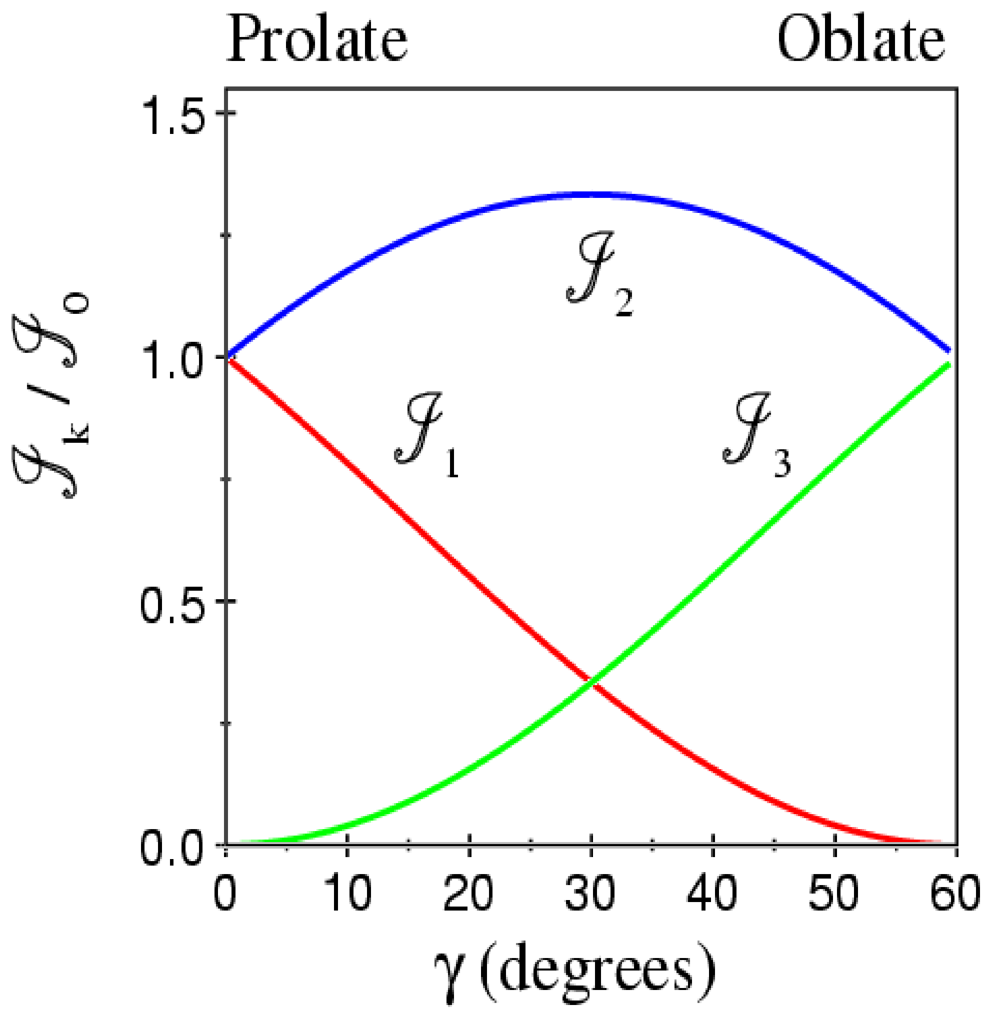
- Spontaneous chiral symmetry breaking can occur in triaxial doubly odd nuclei when there are three mutually perpendicular spin vectors of differing lengths that can form a left-handed or right-handed configuration

Odd-Odd Mass 130 Nuclei

- Region of triaxial shapes ($x \neq y \neq z$)
- Consider the $\pi h_{11/2} \nu h_{11/2}$ configuration

1. The proton Fermi surface lies at the bottom of the $h_{11/2}$ subshell: the proton single-particle j aligns along the short axis
2. The neutron Fermi surface lies at the top of the $h_{11/2}$ subshell: the neutron single-particle j aligns along the long axis
3. The irrotational moment of inertia is largest for $\gamma = 30°$: the core angular momentum aligns along the intermediate axis

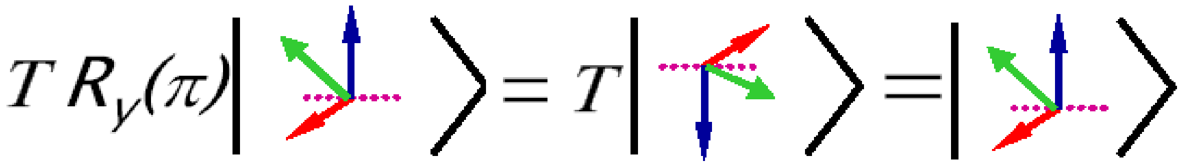
Irrotational Moments of Inertia



- This diagram shows the variation of the moments of inertia $\Im_k$ as a function of the triaxiality parameter γ
- For a prolate nuclear shape (γ = 0°), $\Im_1 = \Im_2$ and $\Im_3 = 0$
- For γ = 30°, $\Im_2$ reaches a maximum and this represents the 'most collective' shape

Chiral Operator

- The chiral operator is a combination of time reversal and rotation by 180°: $\hat{O} = TR_y(\pi)$



- The left-handed and right-handed systems are related to each other by this operator:

$$|L\rangle = \hat{O}|R\rangle \text{ and } |R\rangle = \hat{O}|L\rangle$$

- For a prolate nucleus, chiral symmetry is good: $|R\rangle = |L\rangle$

- However, for the triaxial odd-odd case: $|R\rangle \neq |L\rangle$

Restoration of Chiral Symmetry

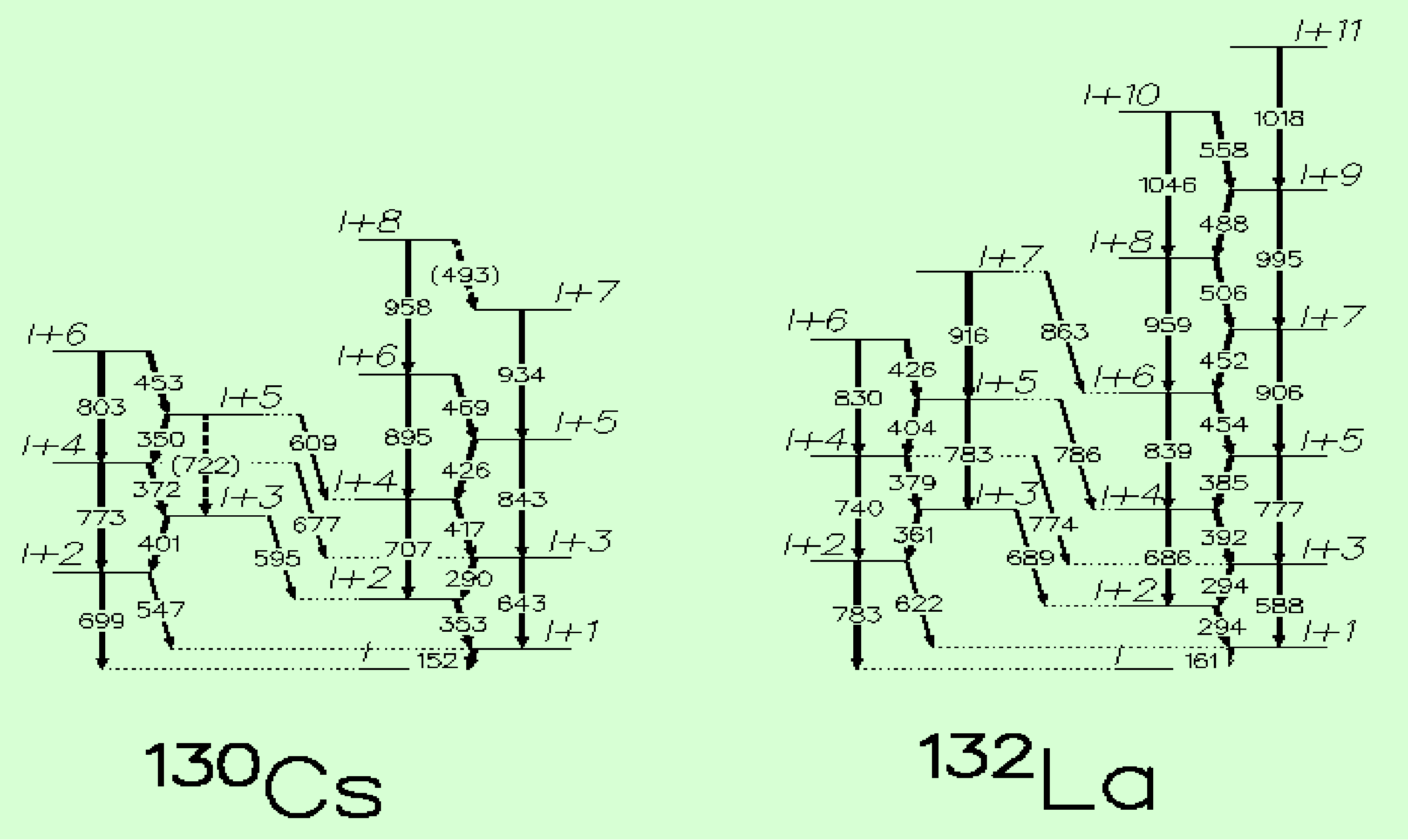
- Note that $|R\rangle$ and $|L\rangle$ are <u>not</u> solutions of the nuclear Hamiltonian in the lab frame and chiral symmetry must be restored by forming wavefunctions of the form (similar to the octupole case):

$$|+\rangle = (1/\sqrt{2})\,[|R\rangle + |L\rangle]$$

$$|-\rangle = (i/\sqrt{2})\,[|R\rangle - |L\rangle]$$

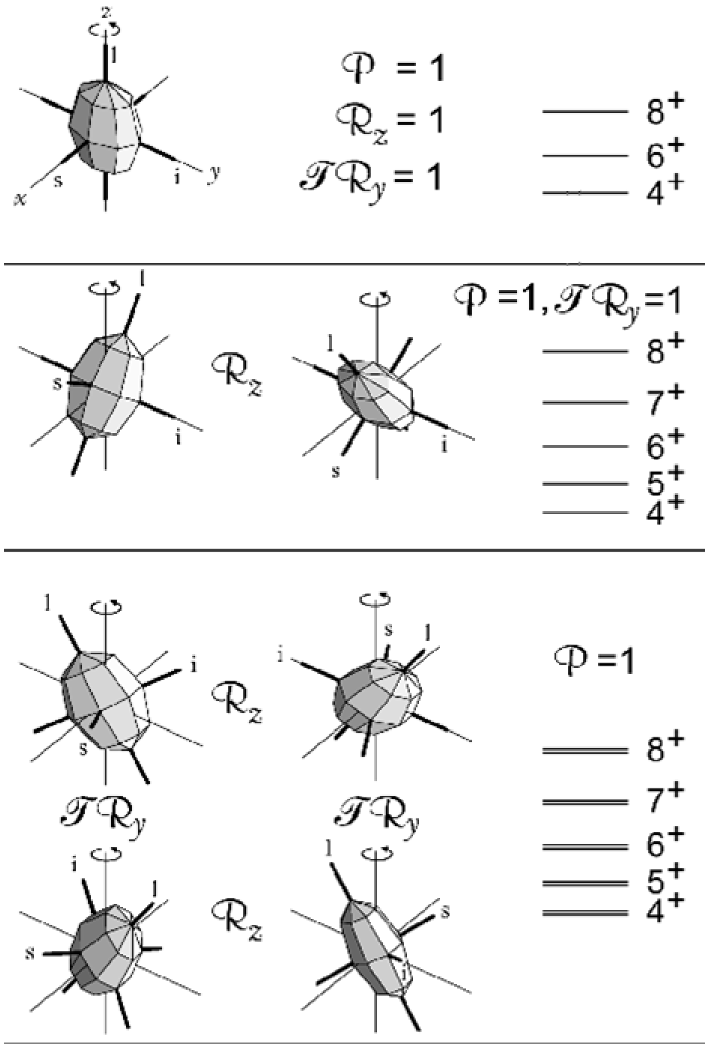
- This leads to the doubling of the states and the occurrence of two (near) degenerate $\Delta I = 1$ bands of the same parity

Chiral Twin Bands



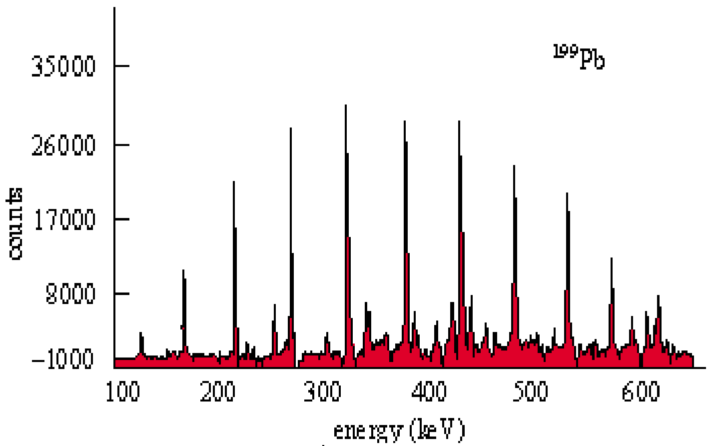
Two near degenerate $\Delta I = 1$ bands of the same parity arise (cf octupole bands: two $\Delta I = 1$ bands of opposite parity)

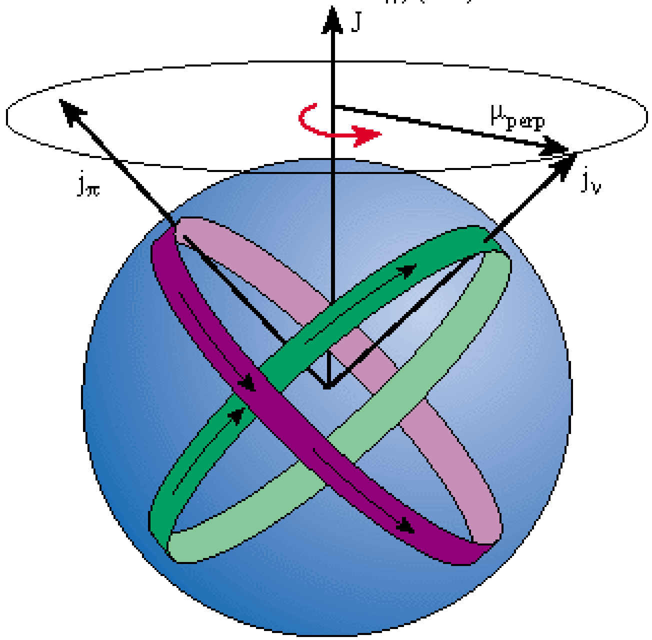
Cranking Symmetries



- If the nuclear spin I lies along one of the principal axes, one $\Delta I = 2$ band arises
- If the spin lies in the plane defined by two principal axes, one $\Delta I = 1$ band arises
- If the spin moves out of these planes, two degenerate $\Delta I = 1$ bands occur (chiral twins)

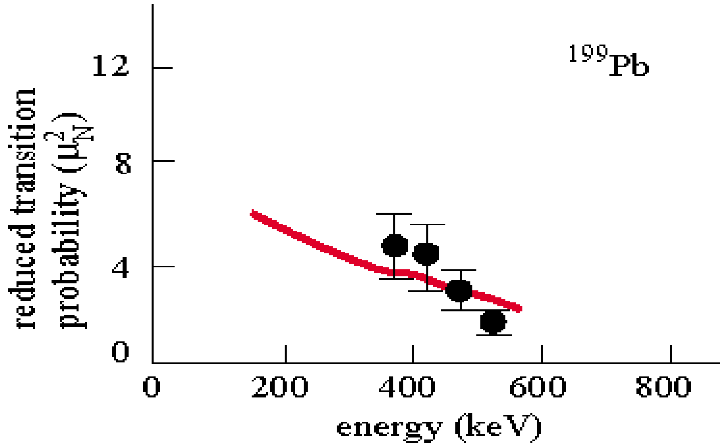
Magnetic Rotation

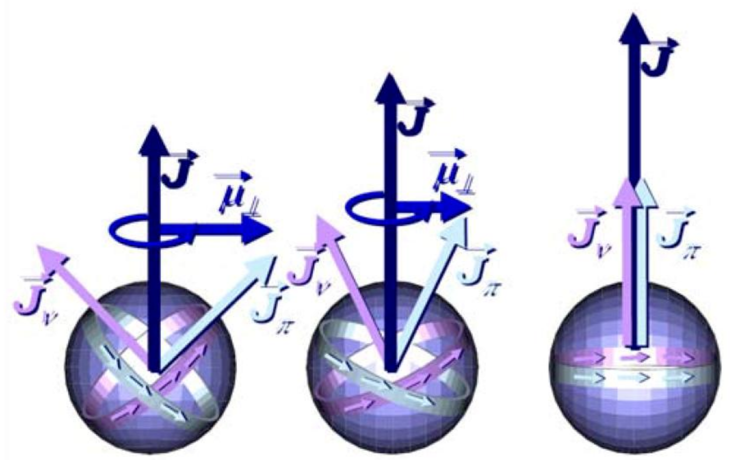




- In spherical lead nuclei, regular bands of intense M1 transitions have been found
- The valence proton and neutron orbitals lie perpendicular to each other and produce a magnetic moment vector that breaks the spherical symmetry of the system and allows 'magnetic' rotation

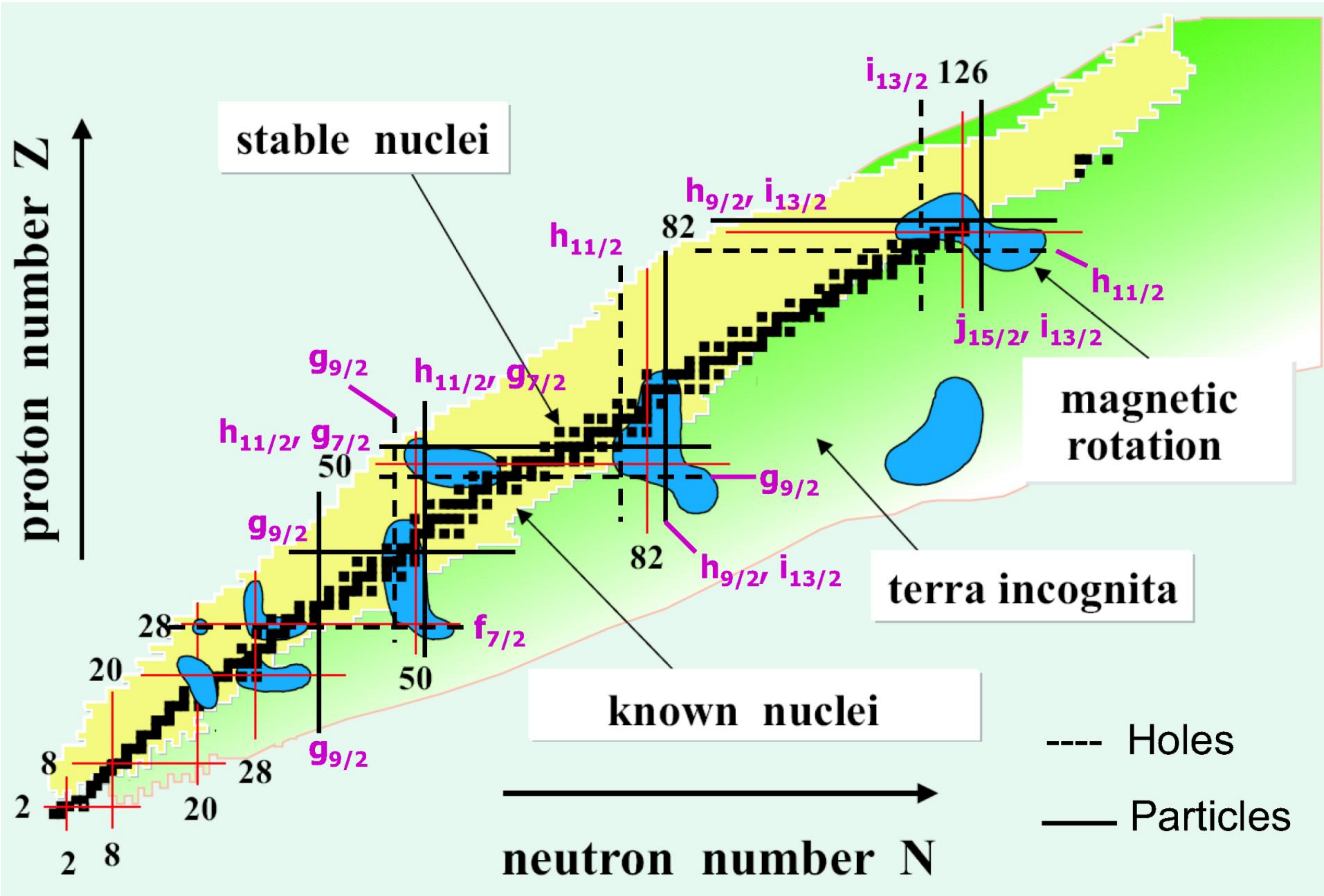
Shears Mechanism



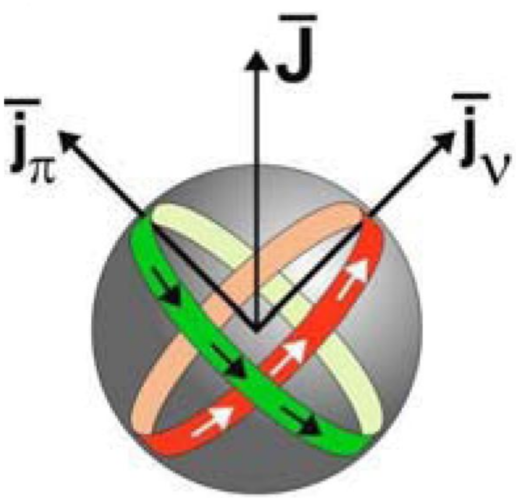


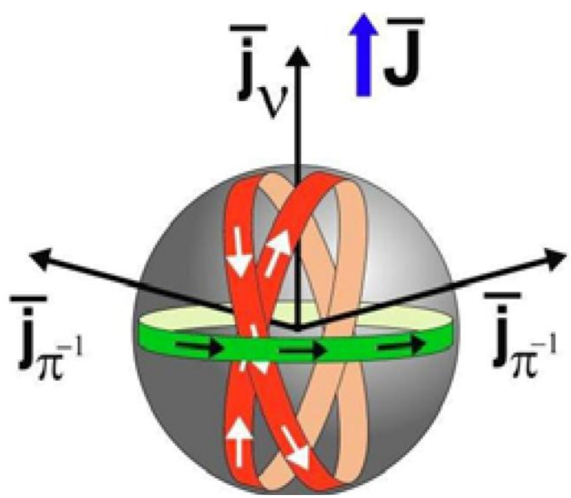
- In magnetic rotation, higher angular momentum is generated by the reorientation of the neutron and proton spin vectors
- Originally perpendicular, the vectors close like the blades of a pair of shears to generate the higher angular momentum states
- The B(M1) strength decreases with increasing spin as $\mu_\perp$ decreases

Shears Systematics



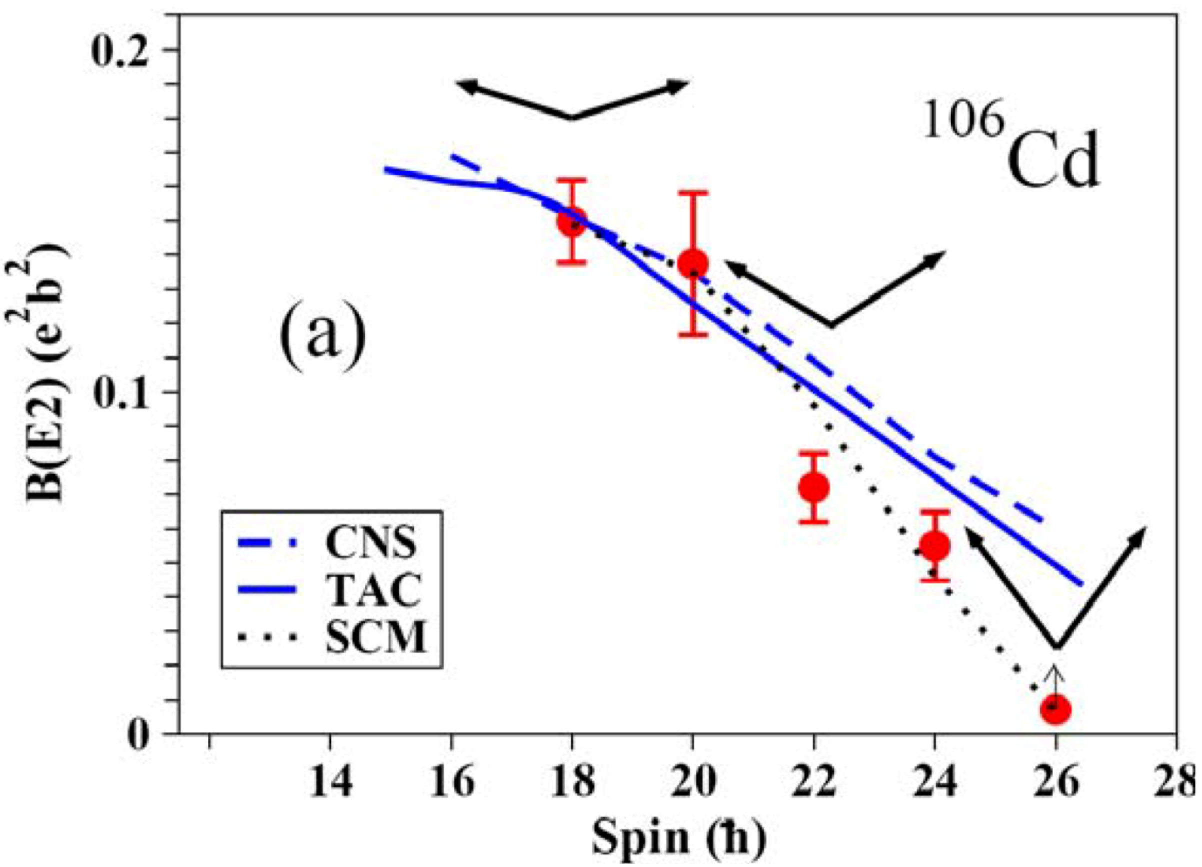
Antimagnetic Rotation





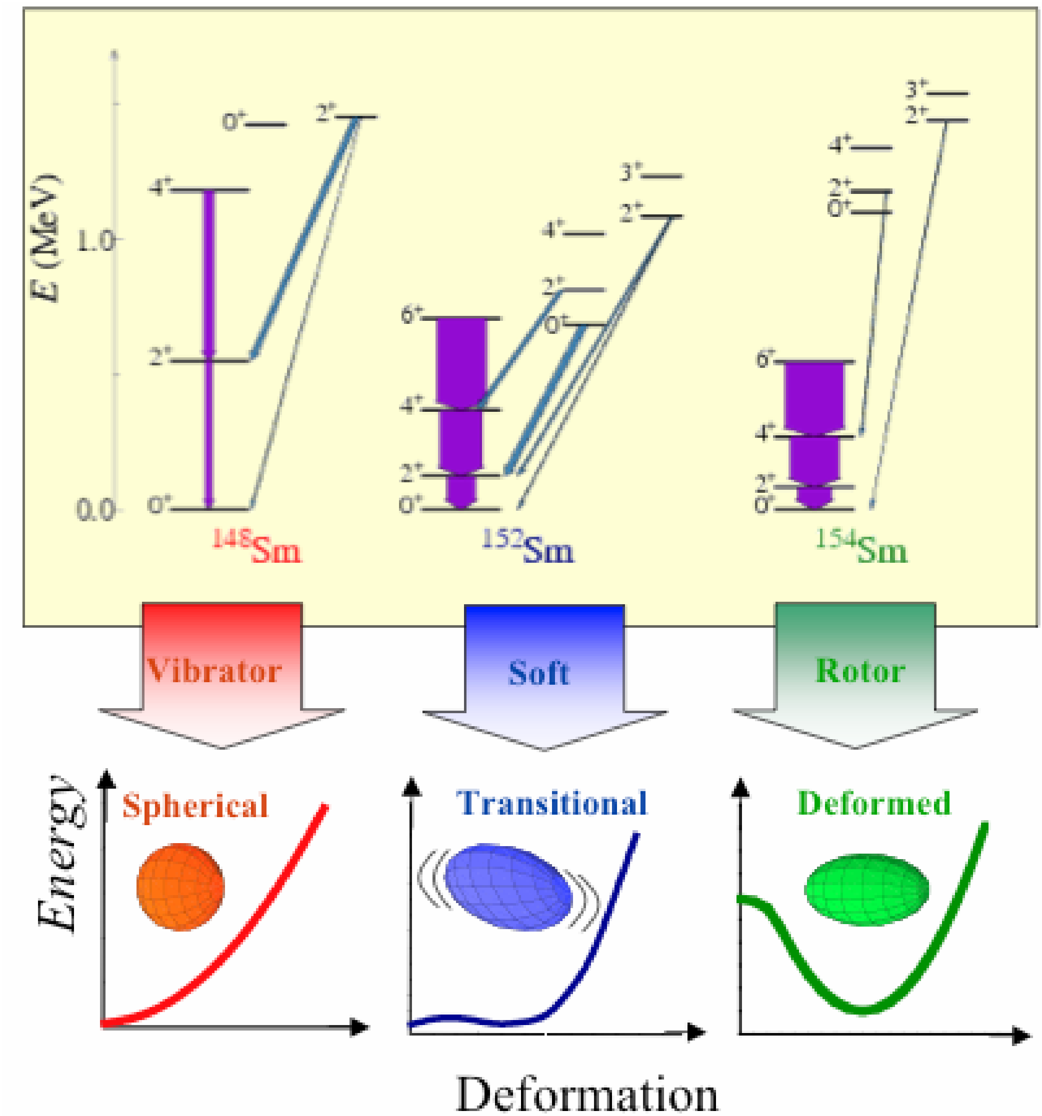
- Expected in weakly deformed nuclei
- In ^{106}Cd the spin is generated by closing the $\pi g_{9/2}^{-1}$ vectors (j_π^{-1} bottom diagram)
- Each $\pi g_{9/2}$ hole combines with one $\nu h_{11/2}$ particle forming a pair of back-to-back shears
- Note that the magnetic moment for this situation is zero, i.e. $\mu_\perp = 0$

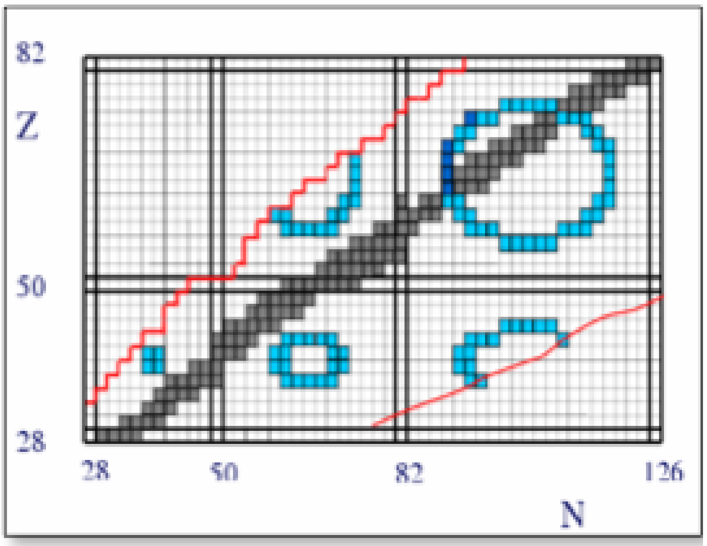
Antimagnetic Rotation in ^{106}Cd



- The yrast band appears to stop at 26^+ with a measured drop in B(E2) values, or collectivity (cf band termination)

Transitional Nuclei



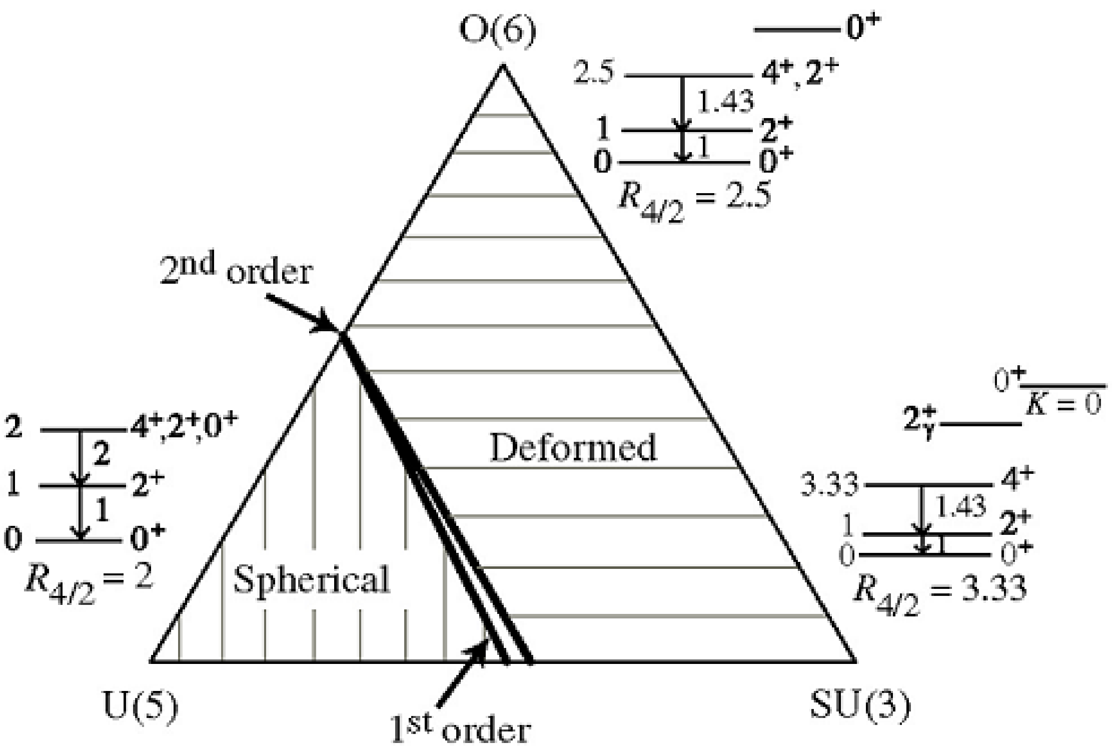


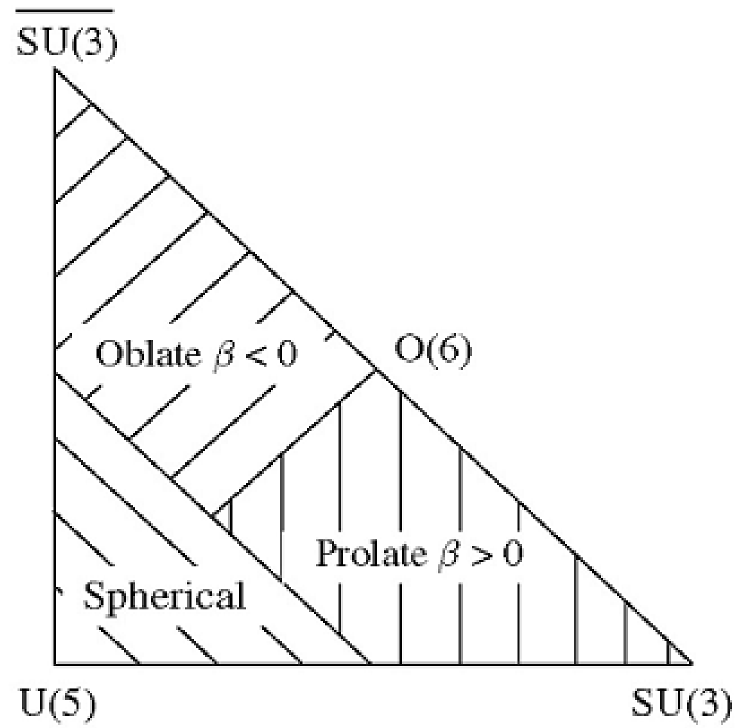
Interacting Boson Model

- Bosons are constructed from fermion pairs
- Nuclear collective excitations are described in terms of N interacting s ($\ell = 0$) and d ($\ell = 2$) bosons
- Algebraic model based on U(6) group
- Limits:
  - SU(3) rotational
  - U(5) vibrational
  - O(6) gamma-unstable

Critical Point Symmetries

gamma soft





vibrator

rotor

- The Casten Triangle